Water Tank Study of the Decay of Trailing Vortices

DIETRICH K. LEZIUS*
NASA Ames Research Center, Moffett Field, Calif.

Underwater towing experiments were carried out with a rectangular airfoil of aspect ratio 5.3 at 4° and 8° angles of attack and at chord-based Reynolds numbers between 2.2×10^5 and 7.5×10^5. Tangential velocity measurements in the downstream region between 100 and 1000 chord lengths indicate rates of vortex decay proportional to $t^{-7/8}$ at 8°, whereas previous flight tests show that the decay rate approaches $t^{-1/2}$ far downstream. The observed behavior is explained in terms of an analytical solution that includes time dependence of the turbulent eddy viscosity, $\nu_T \sim t^m$. It shows that, for $m > 0$, an isolated turbulent vortex decays faster than $t^{-1/2}$. In this case, the decay is accompanied by increasing ν_T, or levels of turbulence, which corresponds to turbulent nonequilibrium flow. The special case of vortex decay with equilibrium flow ($m = 0$) leads to the well-known decay rate $\sim t^{-1/2}$. Since, in towing tank experiments at low Reynolds number, turbulent vortex decay may occur predominantly in nonequilibrium, it is doubtful that such tests correctly predict the late stage of decay of aircraft trailing vortices, when turbulence is the only dissipating mechanism.

Nomenclature

a = core radius
b = wing span
c = wing chord
ΔH = head loss in boundary layer
p = pressure
p_∞ = fluid pressure as $r \to \infty$
$p(0)$ = pressure on the vortex axis
r = radius measured from vortex axis
t = time
Δt = time difference in velocity measurement
T = time parameter, $T = \Gamma_\infty t/c^2$
U_∞ = towing speed
v = tangential velocity component
v_a = tangential velocity at $r = a$
w = axial velocity component
w_0 = axial velocity on vortex axis
z = downstream distance behind airfoil
α = angle of attack
β = constant of proportionality, Eq. (6)
Γ = circulation
Γ_a = circulation at core radius
Γ_∞ = circulation of wing
$\theta, \Delta\theta$ = angle in velocity measurement
ν = kinematic viscosity
ν_T = eddy viscosity
$(\nu_T/\nu)_a$ = eddy viscosity at core radius
ρ = fluid density

Superscripts

m = exponent in expression for time-dependent ν_T, Eq. (7)

Subscripts

a = conditions at the core radius, $r = a$
ν_e = effective viscosity
$(\nu_T/\nu)_0$ = eddy viscosity level after rollup
0 = conditions on the vortex axis, $r = 0$
∞ = conditions at $r = \infty$

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* Research Associate, National Research Council; presently with Lockheed Palo Alto Research Lab., Palo Alto, Calif. Member AIAA.

Introduction

IN several recent wind-tunnel studies (most notably those of Chigier and Corsiglia,[1] Logan,[2] and Mason and Marchman[3]), tangential and axial velocity components of the rolled-up vortex field were measured to downstream distances of $z/c = 30$. The work of Corsiglia et al.[4] extended these measurements to $z/c = 165$, which appears to be a limit imposed by the length of the wind-tunnel test section. Although some initial decay of the maximum tangential velocity is evident in these studies, the rate of vortex decay cannot be established from these data.

The flight measurements of McCormick et al.[5] and Zalovcik and Dunham[6] show that the major portion of the swirl momentum is dissipated at downstream distances larger than 100 chord lengths. In the latter study, it was clearly established that between $z/c = 400$ and $z/c = 2730$, the vortex decay was proportional to $t^{-1/2}$.

No such data have been available for comparison from model towing tests where the mechanics of vortex decay can be studied presumably under well-controlled conditions and without the disturbing effects of wind-tunnel or atmospheric turbulence. The present paper reports on a series of underwater towing experiments in which quantitative velocity measurements were made in the far wake region of a rectangular NACA-0015 airfoil of 0.915-m span with rounded wing tips and aspect ratio 5.33. Chord-based airfoil Reynolds numbers ranged from 2.2×10^5 to 7.5×10^5, and angles of attack were set at 4° and 8°. Use of the hydrogen bubble technique† permitted visual observations of the development of the turbulence structure in the core and in the potential regions during vortex decay. Simultaneously, the flow was photographed on 16-mm movie film from an axial and a crosswise viewing direction. These films yielded quantitative measurements of the tangential velocity profiles and maximum axial velocities. The vortex growth rates measured at the core diameter and the decay rates of the maximum tangential velocity were established as a function of chord-based downstream distance.

The experiments were designed under the original assumption that the mechanics of turbulent, trailing vortex decay are modeled correctly in towing tank experiments. The results, however, showed appreciable deviation from heretofore observed turbulent vortex decay and led to an analytical study of vortex dynamics with time-varying eddy viscosity such that $\nu_T/\nu \sim t^m$. The resulting analytical solutions indicate the existence of a wide

† To mark the flow, hydrogen bubbles are generated by electrolysis on a metal wire.

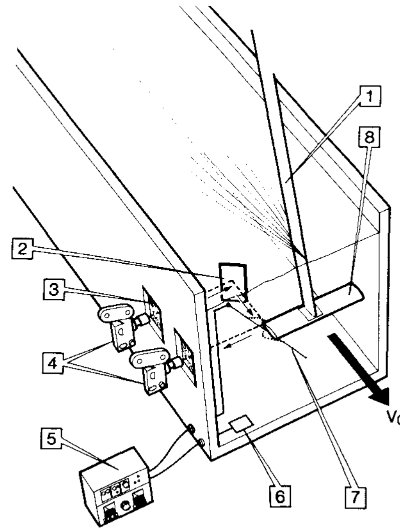

Fig. 1 Experimental setup in towing tank: 1—strut, 2—mirror, 3—window, 4—cameras, 5—pulse generator, 6—ground plate, 7—H_2 bubbles, 8—airfoil.

range of vortex decay rates that depend upon the factor m. Since in the context of the following discussion the eddy viscosity is taken to represent levels of turbulence, the development of the turbulence during vortex decay provides the actual mechanism by which vortex decay rates are established. It appears that, at the low Reynolds numbers obtained in small-scale experiments, the amount of turbulence that is continuously produced by the vortex motion exceeds the amount that can be dissipated by viscosity, thereby raising eddy viscosity levels. The corresponding analytical solutions with $m > 0$ predict a decay rate faster than $t^{-1/2}$ Although inviscid phenomena of trailing vortices such as the Crow instability are less likely to be affected by large differences in scale and Reynolds number, it is suspected that vortex decay in which turbulence mechanisms play a major role is not independent of the Reynolds number. Hence, considerable care must be exercised when relating certain results of small-scale vortex experiments to the mechanics of aircraft trailing vortices.

Experimental Procedure

The towing experiments were conducted in the Lockheed Underwater Missile Facility (4.6 × 4.6 × 55 m). The airfoil was mounted to an overhead carriage by a streamlined strut (see Fig. 1). A hydrogen-bubble generating wire (4.6 μ diam), consisting of three twisted platinum wires, was mounted within the tank at an angle to the vertical axis and below the right wing tip. A 5-kw light source mounted above the tank illuminated the plane of the platinum wire with an intense light sheet of 1.2 × 0.3 m cross section. Two cameras were mounted at airfoil level so that one had a side view of the airfoil; the view of the other camera was pointed in the axial direction by a mirror.

To permit visual studies, the vortex wake was marked by generating hydrogen bubble pulses at the rate of 3 per sec. Excellent photographs of the wake rollup and the subsequent development of its turbulence structure were thus obtained. One such photograph is Fig. 2. Well-defined profiles of the tangential velocity were obtained by generating hydrogen bubble lines at the moment when the settling vortex crossed the electrically pulsed wire. Several such lines are also visible in Fig. 2. The filament of the vortex was usually well marked by entrained hydrogen bubbles that had remained in the water from the previous run. Many of them were individually distinguishable on film and could be used to determine the axial velocity along the vortex centerline.

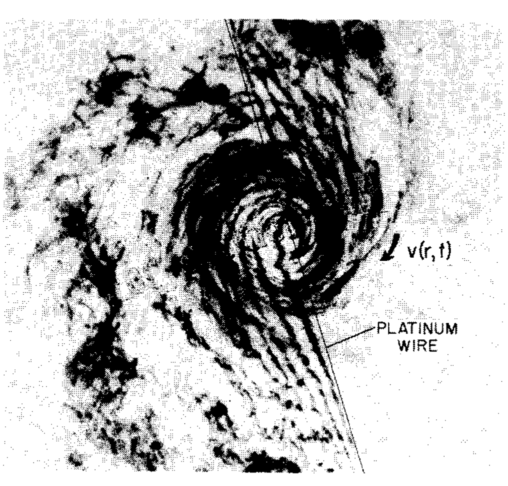

Fig. 2 Axial view of vortex field.

Discussion of Results

Tangential Velocity and Circulation

The method of obtaining velocity profiles by using the hydrogen bubble technique is described in detail by Schraub et al.[7] In the present work, radial distributions of the swirl velocity $v(r)$ were determined by differentiating measurements of the polar coordinates $(r, \Delta\theta)$ of the photographed hydrogen bubble time lines. Based on the time period Δt between electric pulses that were applied to the platinum wire, the tangential velocity profiles were determined from

$$v(r) = r\Delta\theta/\Delta t \tag{1}$$

whereby the downward motion of the vortex system was neglected. After computation of $v(r)$ for each side of the vortex, the results were averaged in order to represent the vortex by one velocity profile.

Figure 3 shows typical velocity profiles that are normalized by the towing speed U_∞ for 4° and 8° angles of attack. The radius is normalized on the wing span b. The velocity profiles decay with downstream distance in a manner quite similar to that for a laminar vortex; this point will be discussed further in a later

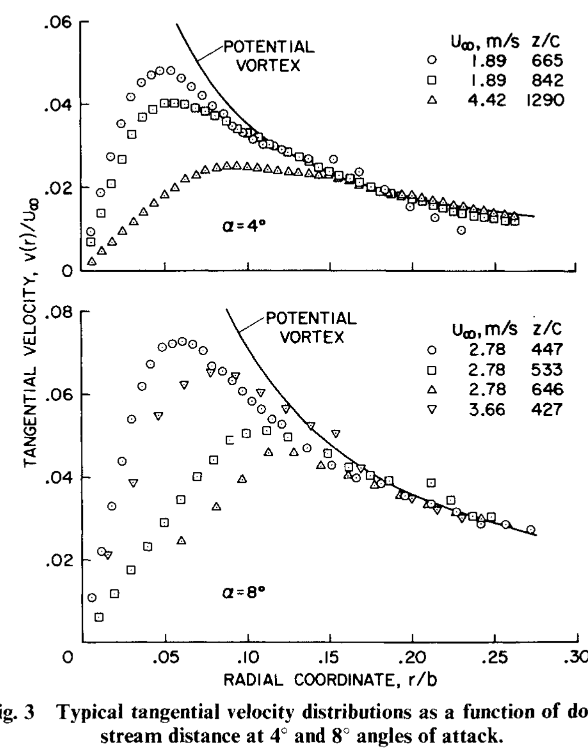

Fig. 3 Typical tangential velocity distributions as a function of downstream distance at 4° and 8° angles of attack.

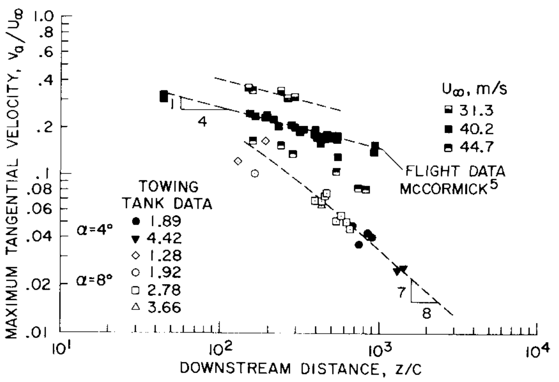

Fig. 4 Decay of maximum tangential velocity.

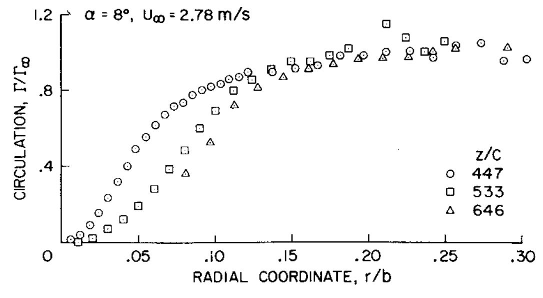

Fig. 6 Decay of circulation profiles with downstream distance.

section. The reduction in maximum tangential velocity proceeds with a simultaneous enlargement of the core. Outside the core, the tangential velocity approaches the envelope of the potential vortex, but it should be noted that, because of viscous or turbulent shear forces, the flowfield decays everywhere simultaneously.

Figure 4 shows the decay of the maximum tangential velocity for the range of downstream distance investigated in this study. For comparison, we also list the flight-test data of McCormick et al.[5] that were normalized appropriately.

Because of insufficient data available from the downstream region at $\alpha = 4°$, the towing tank measurements do not indicate the dependence of v_a/U_∞ upon angle of attack that is exhibited by the flight-test data.‡ For short distances behind the wing, such dependence was established in Refs. 1, 5, and 8. The vortex core measurements shown in Fig. 5, however, exhibit a dependence of vortex growth rate upon the angle of attack. At $\alpha = 8°$, the rate of enlargement of the vortex core is proportional to $t^{7/8}$, which agrees with the rate of vortex decay for the same α in Fig. 4. An apparently slower growth rate of the vortex core $\sim t^{2/3}$ resulted at $\alpha = 4°$; this is in agreement with Brown[9] who considered that the rate of enlargement of the vortex core was due to turbulent diffusion. Nonetheless, the measured rates of vortex dissipation are considerably larger than those measured in flight. They can only be understood when one examines the mechanics of turbulent vortex decay under the influence of time-dependent turbulence effects. Analytical considerations (deferred until a later section) lead to the conclusion that the high rates of vortex dissipation measured here were the results of rising levels of the turbulence, as expressed by the eddy viscosity coefficient.

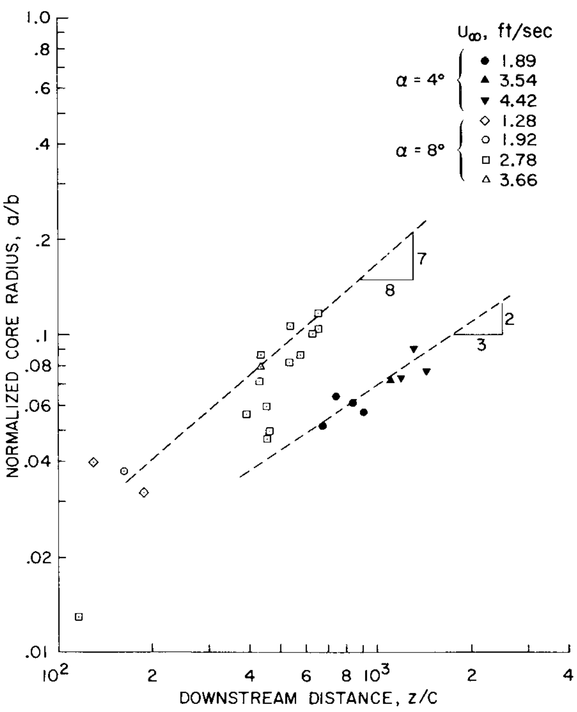

Fig. 5 Growth of core radius behind airfoil.

Typical circulation profiles as a function of downstream distance are shown in Fig. 6. The trend toward self similarity, also evident in the velocity profiles, is well demonstrated by the data. Overshoot of the circulation was observed in some cases and may have been related to radial variations in the rate of turbulent transport of angular momentum, as discussed by Govindraju and Saffman.[10]

The value of the circulation at the location of maximum tangential velocity has been the point of much previous discussion. When the vortex decays under the effect of constant molecular viscosity, Lamb's[11] solution yields the value $\Gamma_a/\Gamma_\infty = 0.715$. In the work of other investigators,[10] the value of Γ_a/Γ_∞ ranges from 0.37 to 0.6, and the model of Govindraju and Saffman[10] allows for $\Gamma_a/\Gamma_\infty = 1.2$. In the recent vortex measurements with a laser velocimeter by Orloff and Grant[8] at $z/c = 2$ behind a rectangular wing, the core circulations varied between 0.2 and $0.8 \times \Gamma_\infty$, depending upon the angle of attack. The wind-tunnel measurements of Corsiglia et al.[4] resulted in $\Gamma_a/\Gamma_\infty \approx 0.4$–0.5. Donaldson's[12] calculations indicate a slight decrease with time of the initial circulation at the core, whereas McCormick et al.[5] proposed that when a decaying vortex maintains geometric similarity, Γ_a/Γ_∞ is constant. This conclusion seems to be confirmed by the data presented herein. Figure 7 shows that the circulation at the core radius remains essentially constant with downstream distance, whereby the large majority of the measurements fall near the theoretical value of $\Gamma_a/\Gamma_\infty = 0.715$. This value also results when the turbulent eddy viscosity ν_T is a constant.

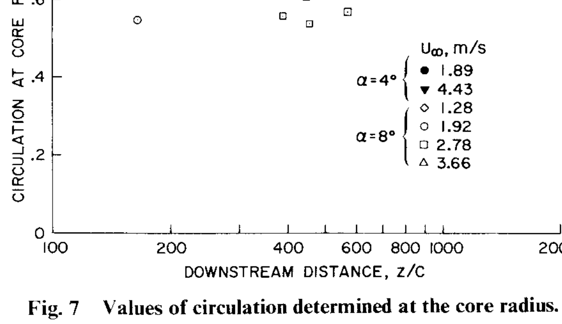

Fig. 7 Values of circulation determined at the core radius.

‡ It is estimated that within the flight data of McCormick et al.[5] the angle of attack varied by a factor of 2.

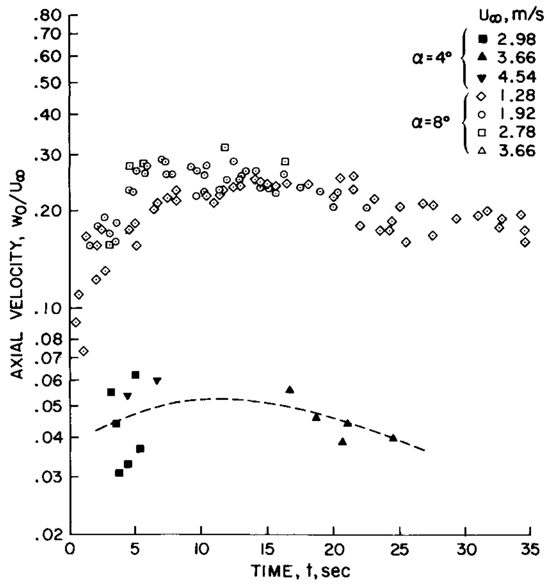

Fig. 8 Maximum axial velocity in towing direction after rollup.

The values plotted in Fig. 7 were determined by two different methods: first, Γ_a/Γ_∞ was obtained directly from plots of Γ/Γ_∞ vs r/b; in the second method, Γ_a/Γ_∞ was determined from the slope of a plot of Γ/Γ_∞ vs $\log(r/b)$, since

$$\left[\frac{d\Gamma(r)/\Gamma_\infty}{d\ln(r/b)}\right]_{r=a} = \frac{\Gamma_a}{\Gamma_\infty} \tag{2}$$

A comparison of these data with calculations of v_T/v from the circulation profiles [see Eq. (13)] indicated that the low values for Γ_a were associated with cases in which v_T/v increased in the radial direction at the location of the core radius. For the remaining cases, v_T/v did not depend on r.

Axial Velocity

The importance of the axial core velocity has been previously discussed by Batchelor,[13] Brown,[9] and Moore and Saffman.[14] As a result of these studies, it has become evident that the axial velocity in the core can be directed either toward the wing or away from it. The direction of flow depends upon a balance between the viscous airfoil drag, which causes a wake to form, and the swirl-induced radial pressure gradient, which tends to propel the core fluid as a jet down the vortex core. Batchelor's formulation for the radial variation of the axial velocity, if expressed in coordinates of a stationary observer within a fluid at rest, is given by

$$w(r) = \left[U_\infty{}^2 + \frac{1}{4\pi^2}\int_r^\infty \frac{1}{r^2}\frac{\partial\Gamma^2(r)}{\partial r}dr - 2\Delta H\right]^{1/2} - U_\infty \tag{3}$$

where ΔH is the head loss due to the viscous boundary layer on the wing. If the viscous drag term is larger than the integral term, as is the case at moderate angles of attack, one observes a velocity defect in the core, $w < 0$. Nevertheless, the data of Chigier and Corsiglia[1] show that, when the angle of attack increases from 8° to 12°, there is a reversal from a wake flow in the core to an axial velocity excess, which was measured at a downstream distance of $z/c = 9$. Hence, at large angles of attack, the swirl-induced pressure drop along the vortex axis immediately behind the wing is strong enough to accelerate fluid from the wing-tip area in the downstream direction, despite the momentum loss incurred by passing through the viscous boundary layer.

In the present experiments, quantitative axial velocity profiles were difficult to obtain, but the visual studies revealed the presence of velocity defects in the vortex cores at 4° and 8° angles of attack. The most notable feature of the axial flow was that the velocity reached a sharp radial peak near the vortex axis. This observation qualitatively agrees with the singular nature of expression (3) at $r = 0$ and with theoretical results of Brown[9] regarding the rollup of the momentum deficient boundary-layer fluid into the vortex. According to that analysis, the axial velocity profile, whether it represents a wake or a jet, assumes a $1/r$ dependence through the core.

The measurements of the axial velocity on the vortex axis reported here were made by following fine lines of entrained hydrogen bubbles which marked the vortex filaments in many cases. As shown in Fig. 8, the magnitude of the maximum defect velocity depended strongly upon the angle of attack. Furthermore, these measurements revealed that the velocity on the vortex axis accelerated to a maximum before decaying with time. So far, this behavior has not been treated by any of the theories dealing with the axial core flow. The mechanism for the initial acceleration may be found in the positive pressure gradient in the downstream direction, which is caused by the decay of the integral for the radial pressure drop

$$p(0) - p_\infty = -\rho\int_0^\infty \frac{v^2(r)}{r}dr \tag{4}$$

Since, in stationary coordinates, the tangential velocity decays with time, we have for the local pressure gradient on the axis:

$$\frac{\partial p(0)}{\partial z} = -\rho\frac{\partial}{\partial z}\int_0^\infty \frac{v^2(r)}{r}dr = -\frac{\rho}{U_\infty}\int_0^\infty \frac{1}{r}\frac{\partial v^2}{\partial t}dr > 0 \tag{5}$$

where we made the transformation $z = U_\infty t$. The fluid surrounding the axis would thus tend to accelerate toward the low-pressure region behind the wing. Eventually, viscous and turbulent friction cause decay of the axial flow component; the initial phase of this decay is shown in Fig. 8.

Effect of Turbulent Viscosity on Vortex Decay

Theoretical Considerations

It is generally agreed that the turbulent eddy viscosity v_T of the vortex flow is considerably higher than the molecular viscosity. According to data from various investigators collected by Owen,[15] estimates for v_T/v fall between 30 and 2000, depending upon the vortex Reynolds number Γ_∞/v. Kuhn and Nielsen[16] proposed a theory to calculate the vortex decay by using the z-component of the eddy viscosity evaluated at the vortex axis. Their predictions fit the wind-tunnel data of Chigier and Corsiglia[1] for values of the eddy viscosity ratio between 200 and 450. Squire[17] suggested that Lamb's laminar solution can still be applied to the turbulent vortex decay when the eddy viscosity is a constant, given by

$$v_T = \beta\Gamma_\infty \tag{6}$$

Owen proposed a more sophisticated model, but in order to achieve agreement with experiment, also finally assumed that v_T is proportional to Γ_∞. Both models result in a rate of decay of v_a proportional to $t^{-1/2}$.

The flight measurements of McCormick and the data of this study show, however, that vortex decay rates other than $t^{-1/2}$ are possible. This would mean that the eddy viscosity, which is directly proportional to the intensity of turbulent fluctuations in the shear flow, may vary with time. Since in towing experiments the airfoil is moved through an undisturbed fluid, the initial rate of generation of turbulent fluctuations by the shearing motion within the vortex outweighs the rate of viscous dissipation. Hence, one would expect an initial increase in the eddy viscosity and a corresponding contribution to the rate of vortex decay. This sort of reasoning is supported by the visual observations of the present study, which indicate that the rolled-up airfoil boundary layer was not fully turbulent and does not account for the higher turbulence levels that are present in the vortex field at some later time. We are thus dealing with a nonequilibrium flow in which the nonequilibrium is constituted by the imbalance between turbulence production and dissipation. In the work of Baldwin et al.,[18] for instance, nonequilibrium conditions were taken into account in order to calculate the downstream decay of turbulent trailing vortices. Significantly, the initial levels of turbulence used in these computations were comparable in their magnitude to values that are known from flight and wind-tunnel tests, and were considerably larger than those that can be sustained by a vortex flow. Because turbulence dissipation was greater than production initially, the calculations showed a rapid decrease of the eddy viscosity ratio, during which period the circumferential velocity attained only a very small decay rate. When a constant viscosity ratio of $(v_T/v)_a \approx 150$ was established, the vortex decay rate approached $t^{-1/2}$.

Having thus, at least qualitatively, established that the vortex decay rate depends upon the development of turbulence, we next consider an analytical solution for the decay of what is at first a potentially turbulent vortex when the eddy viscosity is allowed to vary with time but remains uniform in the radial direction. Since the actual development of turbulence in the vortex flow is not considered, the solution shows the dependence of the rate of vortex decay upon particular assumed variations in $v_T(t)$, some of which are actually observed.

We define the effective viscosity of the flow as

$$v_e \equiv v + v(v_T/v)_0\, T^m = v[1+(v_T/v)_0\, T^m] \tag{7}$$

where $(v_T/v)_0$ is the initial eddy viscosity level in the flow after rollup. The symbol T is dimensionless time for appropriate parameters of the flow, e.g., $T = t\Gamma_\infty/c^2$. In order to solve the resulting differential equation, we make the further assumption that $(v_T/v)_0\, T^m \gg 1$. This assumption is justified in wind-tunnel and flight tests, except perhaps in towing experiments at very low Reynolds number. With this simplification, the two-dimensional vortex motion is governed by

$$\frac{\partial v}{\partial t} = v\left(\frac{v_T}{v}\right)_0 \left(\frac{\Gamma_\infty t}{c^2}\right)^m \left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right] \tag{8}$$

The solution to this equation is

$$v(r,t) = \frac{\Gamma_\infty}{2\pi r}\left\{1 - \exp\left[-\frac{(m+1)r^2}{4v(v_T/v)_0(\Gamma_\infty t/c^2)^m t}\right]\right\} \tag{9}$$

We note that the over-all features of the velocity profile remain the same as with constant v_T; but the time rate of change of $v(r,t)$ now depends upon the development of the eddy viscosity through the factor m. We want to establish the rate of change of the maximum circumferential velocity and the growth of the vortex core as m assumes various values. Solving for the radius at the point of maximum tangential velocity $a(t)$ and substituting $r = a(t)$ in Eq. (9), we have

$$v(a,t) \sim U_\infty(\Gamma_\infty t/c^2)^{-(m+1)/2} \tag{10}$$

and

$$a(t) \sim b(\Gamma_\infty t/c^2)^{(m+1)/2} \tag{11}$$

In contrast to vortex motion with constant v_T, the growth of the vortex is proportional to $t^{(m+1)/2}$, i.e., time dependent changes in v_T contribute to the rate of vortex decay through the additional exponential factor $m/2$. Table 1 demonstrates the significance of these results.

Surveying the literature, including the results of this study, one finds that the range of time decay factors m listed in Table 1 includes those either previously measured or predicted using various theories of turbulent shear flow. The value of m can be regarded as a quantitative measure for the initial degree of non-equilibrium in the turbulent vortex flow after rollup. The case $m \to -1$ is apparently characteristic of the initial decay phase of trailing airplane vortices, which show no appreciable decay within the downstream range of $z/c < 100$. The initial value of m,

Table 1 Time dependence of maximum circumferential velocity and core radius upon assumed time variation of v_T/v

m	-1	$-2/3$	$-1/3$	0	$1/3$	$2/3$	1
$v(a,t)$	~ 1	$\sim t^{-1/6}$	$\sim t^{-1/3}$	$\sim t^{-1/2}$	$\sim t^{-2/3}$	$\sim t^{-5/6}$	$\sim t^{-1}$
$a(t)$	~ 1	$\sim t^{1/6}$	$\sim t^{1/3}$	$\sim t^{1/2}$	$\sim t^{2/3}$	$\sim t^{5/6}$	$\sim t$

whether it be positive or negative, does not remain constant but tends toward zero at the same rate at which equilibrium conditions are established in the vortex flow. When, on the other hand, the initial conditions of the vortex correspond to those of an equilibrium flow, the $t^{-1/2}$ dependence occurs from the outset as shown in the calculations of Baldwin et al. The authors call these solutions self-similar, but we recognize that all solutions with constant m possess the self-similar property. Hence, $m = 0$ represents the law for turbulent vortex decay when equilibrium conditions have been established in the flow. The available data on wake vortices suggest strongly that the time required to reach this condition is proportional to the degree to which the initial flow conditions deviated from those of equilibrium.

Experimental Results

In view of the previous analytical findings, our test results in a towing tank suggest primarily that the turbulence intensities, and hence eddy viscosity levels, were rising during the decay of the vortex, thereby resulting in faster decay rates than had been previously observed. In fact, using the slopes of $\mp\frac{7}{8}$ in Figs. 4 and 5, respectively, and the exponents of Eqs. (10) or (11), we obtain $m = \frac{3}{4}$, and hence $(v_T/v)_a \sim t^{3/4}$, for the data at 8°. Figure 5 suggests that, $\alpha = 4°$, the vortex decayed somewhat slower ($m \approx \frac{1}{3}$), although this trend is not indicated in Fig. 5 because of the insufficient downstream range of the data. It is significant, however, that despite increasing turbulence intensity, the equilibrium level was not reached within the lifespan of the vortex, since this would have resulted in a final dependence as $t^{-1/2}$. Using the appropriate equations given in the papers of Baldwin et al.[18] and Donaldson,[12] the equilibrium eddy-viscosity level was estimated to be $(v_T/v)_{\text{equ.}} \approx 150$.

Distributions of v_T/v with radius and time were computed from graphical fits to the circulation profiles. Substituting

$$v_T/v = \left(\frac{v_T}{v}\right)_0 \left(\frac{\Gamma_\infty t}{c^2}\right)^m \tag{12}$$

into Eq. (9) and solving for v_T/v, we have

$$\frac{v_T(t)}{v} = -\frac{(m+1)r^2}{4vt\ln\{[\Gamma_\infty - \Gamma(r,t)]/\Gamma_\infty\}} \tag{13}$$

Using $m = \frac{3}{4}$ as established from the data of Fig. 4, v_T/v profiles computed from Eq. (13) were accepted as representing levels of the eddy viscosity approximately when v_T/v was invariant with r. Surprisingly, this condition was satisfied by the majority of the examined vortex profiles. Some typical results are shown in Fig. 9, which indicate additional dependence of v_T/v upon Γ_∞ through

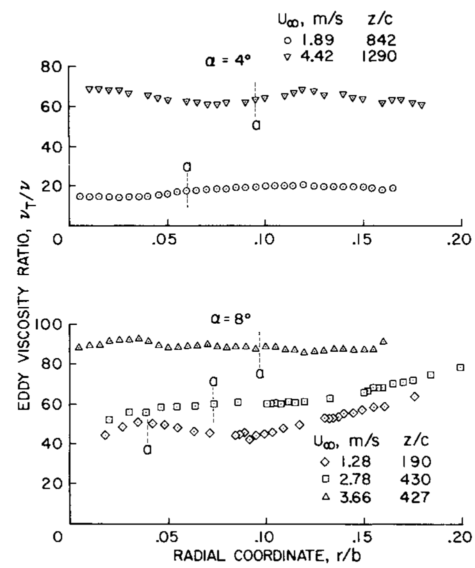

Fig. 9 Eddy viscosity profiles computed from experimental data and Eq. (13).

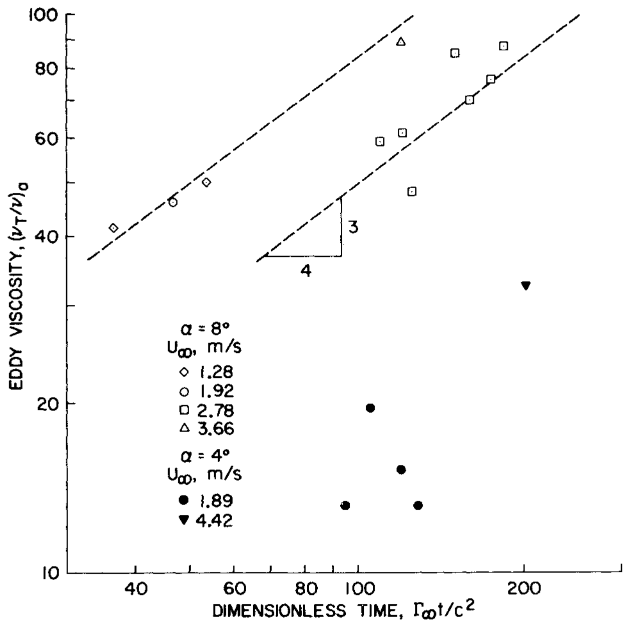

Fig. 10 Time dependence of eddy viscosity determined from data.

the combined effects of α and U_∞. Apparently, the eddy viscosity, does not vary appreciably between the inner core and a radial distance corresponding to about three core radii. Thus, one can argue that the value of ν_T/ν determined at the core radius is a fair approximation of the effective turbulent viscosity of the vortex. Figure 10 shows values of $(\nu_T/\nu)_a$ plotted against $\Gamma_\infty t/c^2$ for $\alpha = 4°$ and $8°$. The data belonging to $\alpha = 8°$ demonstrate rather convincingly that in these experiments the eddy viscosity increased with time. The slope of approximately $\frac{3}{4}$ of the band that envelops the data points agrees with the results of Figs. 4 and 5. The width of this data band may be regarded as an indication of the accuracy of the experimental method, whereby initial turbulence, left in the water from the previous run could have had an effect upon the measurements. Although the data at $\alpha = 4°$ are too sparse to indicate the rate at which the eddy viscosity increased with time ($m \approx \frac{1}{3}$ is suggested by Fig. 5), they show considerable dependence of turbulence levels upon α for comparable U_∞ (compare points for $U_\infty = 1.89$ and $U_\infty = 1.92$ m/sec). Indeed, a two-fold dependence is suggested by the data, such that both the initial eddy viscosity level after rollup, here

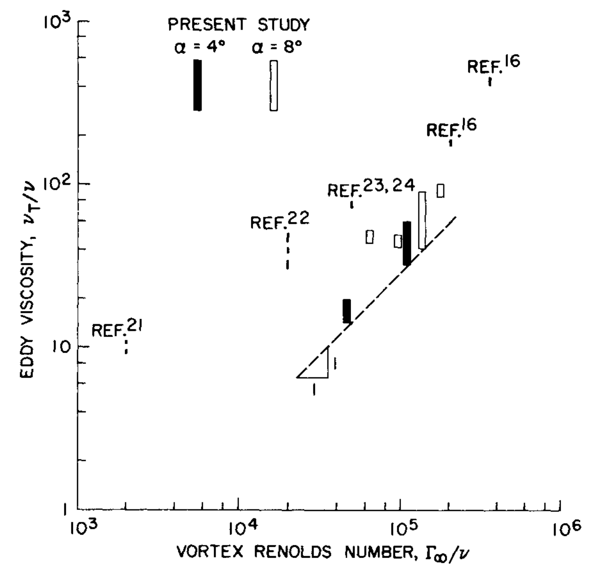

Fig. 11 Comparison of experimental eddy viscosity data.

defined as $(\nu_T/\nu)_0$, and the subsequent rate of increase depend upon Γ_∞ through α and U_∞. Such dependence is entirely expected. The initial intensity of turbulent fluctuations that are associated with airfoil drag and the magnitude of the shearing motions that result from the induced circulation are proportional to angle of attack and towing speed.

A summary of the eddy viscosity data as a function of the vortex Reynolds number Γ_∞/ν is presented in Fig. 11, together with the wind-tunnel data of Refs. 16 and 21–24. The lower eddy-viscosity levels obtained in the water tank experiments are attributed to the absence of the turbulence that usually accompanies wind-tunnel tests. At high Reynolds numbers, on the other hand, the relationship $\nu_T/\nu \sim \Gamma_\infty/\nu$ proposed by Owen[15] is supported by the wind-tunnel as well as by the towing-tank data.

Conclusions

The experimental and analytical results of this study show that the mechanics of turbulent vortex decay depend upon the Reynolds number through the levels of turbulence that are introduced into the vortex. Whereas initial turbulence intensities of the rolled-up vortex reflect those generated within the airfoil boundary layer, the subsequent development of the turbulence depends largely upon the magnitude of the shearing motions within the vortex. These are introduced as a result of the generation of lift and depend on Reynolds number and angle of attack. An imbalance between the initial turbulence levels and those that can be maintained by the vortex motion causes the vortex to decay in nonequilibrium conditions which affect the rate of vortex decay. An initial deficit leads to generation of more turbulence while an initial excess of turbulence in the rolled-up boundary layer may decay toward an equilibrium level.

An analytical solution shows that when nonequilibrium conditions are expressed by time varying eddy viscosity levels, similarity solutions are obtained that allow vortex decay rates other than the $t^{-1/2}$ dependence. Analysis of the experimental data of the present study in the light of these solutions indicates that the decay rate according to $t^{-7/8}$ resulted from an increase of the eddy viscosity with time, apparently a direct result of the low airfoil Reynolds numbers. The solution shows further that only under equilibrium conditions (ν_T/ν = const) does the self-similar solution for vortex decay depend upon $t^{-1/2}$. Although flight data as well as flow calculations have established this dependence as the expected long-time behavior, it appears that at the Reynolds numbers prevailing in flight the turbulence of the rolled-up boundary layer cannot be maintained by the free shear flow of the vortex, resulting in diminishing turbulence and hence a slower decay rate until equilibrium conditions are reached.

Since in towing experiments at low Reynolds number, the vortex may decay entirely in the nonequilibrium condition, such results must be interpreted with some care. For instance, the low turbulence levels generated by the airfoil in our tests resulted in conditions sufficiently far removed from equilibrium that the vortex could be considered decayed before it reached the equilibrium stage.

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Turbulent Boundary-Layer Flow Separation Measurements Using Holographic Interferometry

ALBERT G. HAVENER* AND ROGER J. RADLEY JR.†
Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio

Unique experimental data are obtained from optical measurements made of a separated turbulent boundary in a supersonic, high Reynolds number flow. Holographic interferometry is used to measure density in a boundary layer that is separated from a flat surface Separation is caused by turning the boundary layer through a strong compression turn. A rational method is developed to calculate velocity distributions from the optical data with assumed conditions for the total temperature and static pressure. Also, an array of double pulse holographic interferograms is used to define qualitatively the transient behavior of a separated flow.

Introduction

HOLOGRAPHY has permitted greater flexibility in wind-tunnel testing than previously used optical techniques because the image reconstructed from the hologram contains all the information of the original density field. This means the reconstructed image can be analyzed using conventional processes. For example, shadowgrams can be obtained by focusing on different planes within the image of the density field, numerous schlieren photographs can be made simply by varying the knife edge orientation in the reconstructed image of the light source, and interferometric measurements can be made by combining the reconstructed light waves describing the flowfield with a set of reference light waves that are independent of the flowfield.

The chief motivation for this project is the successful results recently obtained from holographic interferometric measurements made of a thin, supersonic turbulent boundary layer attached to a flat surface.[1] The development of holographic wind-tunnel techniques has progressed steadily for the past few years, and the success of the attached boundary-layer investigation further demonstrates the accuracy, reliability, and capability of this flow measuring technique.[1,2]

The primary effort of this study was to use the holographic system to measure density variations in a separated boundary layer. In addition, an effort was made to calculate velocity from the density measurements. This calculation is realistic for the attached boundary layer because the total temperature and static pressure can be assumed with reasonable certainty. In the current investigation it is not obvious how to calculate velocity from density because the exact conditions for the total temperature and static pressure are unknown. Consequently, a new

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Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.

* Captain, U.S. Air Force, Hypersonic Research Laboratory; presently Det OPNL, Sacramento Air Materiel Area, Air Force Logistics Command, McClellan Air Force Base, Calif.

† Captain, U.S. Air Force, Hypersonic Research Laboratory; presently Associate Research Engineer, United Aircraft Research Laboratories, East Hartford, Conn. Member AIAA.